647510 **MSC3MatC304x** **Seat No :_____**

M.Sc.(Maths.) Semester - 3 (*CBCS*) Examination
Oct/Nov-2022 (NEW COURSE)
Discrete Mathematics (Core (New))

Time: 2:30 Hours **Marks: 70**

Instructions:

1. All questions are compulsory.
2. Figures to the right indicate marks.

Q.1 (a) **4 Marks**

State and Prove Fundamental Theorem of Homomorphism of groups.

Q.1 (b) **Answer any two** **10 Marks**

(1) Define Quotient Group. Prove that $^{Z}/_{2Z}$ is quotient group. 5

(2) Define Transitive Closure. State and Prove necessary and sufficient condition for Transitive closure. 5

(3) Prove that every monoid has Unique identity and unique inverse. 5

Q.2 (a) **4 Marks**

Define Homomorphism of lattices. Prove that (S_8, D) & $(A, \leq)$ are homomorphic. Where A ={1,2,3,4}

Q.2 (b) **Answer any two** **10 Marks**

(1) Using K map minimize the following Boolean Expression 5

$$\alpha\,(x, y, z, t) = x\,y' + x\,y\,t + x't' + x'y\,z\,t$$

(2) Define Boolean Function. Find the value of α (0,0,1) for the Boolean expression $\alpha\,(x, y, z) = \Sigma$ (1,2,4,6) 5

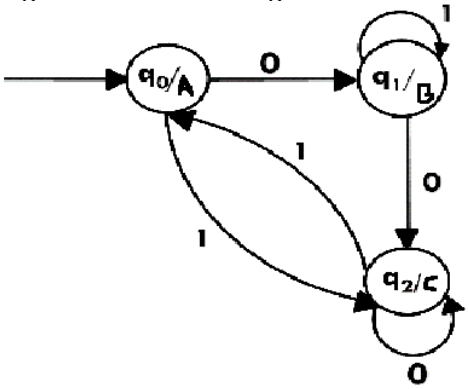
(3) Express the following as equivalent of PoS & SoP Canonical Form for Boolean Expression $\alpha\,(x, y, z) = x' + y\,z'$ 5

Q.3 (a) **4 Marks**

State and Prove Pumping Lemma.

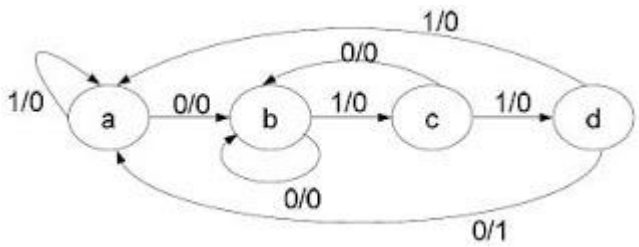
Q.3 (b) **Answer any two.** **10 Marks**

(1) Define Moore machine. Construct transition table and find the output of the string 010 in the following Moore machine. 5



(2) What is language? Write down operations on languages. 5

(3) FSM M is given by transition diagram. 5
(1) Determine the machine (2) Construct input –output table. Check whether the string is accepted by the machine? String: 1011.



Q.4 (a) **4 Marks**

Define Quantifiers. State & explain the rules of inference.

Q.4 (b) **Answer any two** **10 Marks**

(1) Define tautology. Prove the following statement formula is tautology ,using truth table: $(p \rightarrow (q \rightarrow r) \equiv (p \wedge q) \rightarrow r$ 5

(2) Define Statement. Check the validity of the argument using logical connectives. "If two sides of a triangle are equal, then the opposite angles are equal. Two sides of a triangle are not equal.The opposite angles are not equal." 5

(3) Define n-place predicate. Check the validity of the argument using predicate and quantifiers. "Everyone who knows english well then get a good job. Ram did not get the good job.So he did not know English well." 5

Q.5 (a) **4 Marks**

Suppose that G is an $n \times k$ standard generator matrix. Then *Show that* $C = \{y : Gx = y \text{ for } x \in Z_2^K\}$ is an *(n,k)*-block group code.

Q.5 (b) **Answer any two** **10 Marks**

(1) Let C be Binary(6,3) code with parity check matrix 5

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 0 & 1 & 1 & 1 \end{bmatrix}$$

(1) Find the cost leaders and their syndromes.

(2) Use syndrome decoding to decode the received vector: 110000

(2) Let C be the binary linear code with generator matrix 5

$$H = \begin{bmatrix} 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 1 \end{bmatrix}$$

Write down a parity check matrix H for C. Explain how the minimum distance of C may be deduced from H. Find d(C).

(3) Find the generator matrix of C. The parity check matrix is simply obtained from H as, 5

$$G = [A^T \cdot I_3] = \begin{bmatrix} 0 & 1 & 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

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